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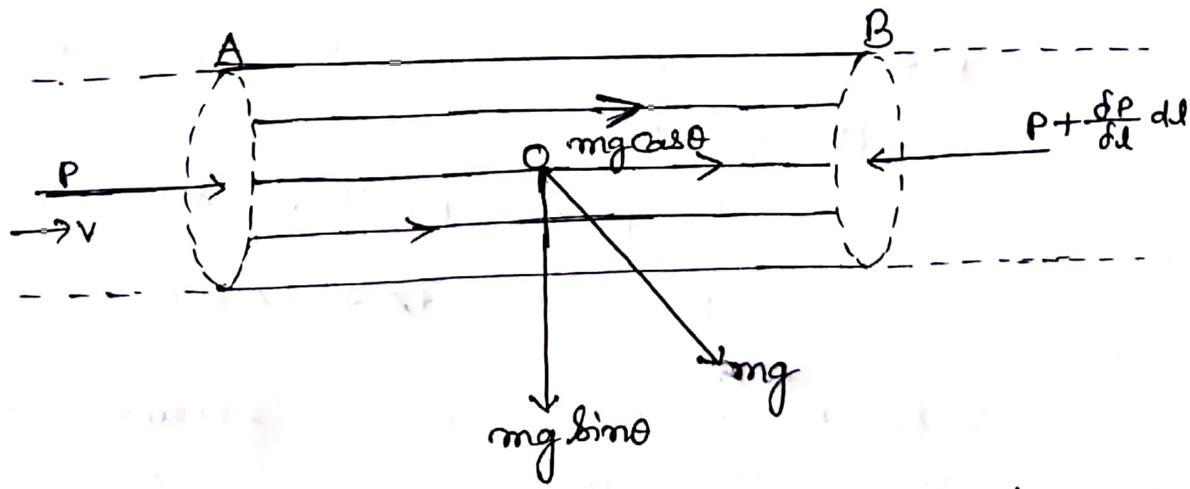
Euler's Equation for perfect fluid.

- Perfect fluid :- A perfect or Ideal fluid is one which is incompressible, homogeneous and frictionless and which cannot sustain any tangential force or action in the form of shear (i.e. coefficient of rigidity = 0) but the normal force acts between the adjoining layer of it. The pressure at any point of a perfect fluid is equal in all direction whether the fluid be at rest or in motion. This theory defines some concepts of the flow such as wave motion, the lift and induced drag of an airfoil etc. but fails to define the phenomena as for example skin friction, drag of body etc.

Euler's equation is an Important equation applicable to both steady and unsteady flow.



Let an infinitesimally small portion AB of a tube of flow of length dl and area of cross section dA. If ρ is the density of the incompressible fluid then

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Mass of the fluid in portion AB

$$m = dA\,dl\,\rho$$

and weight(mg) $= dA\cdot dl\,\rho g$

mg acts vertically downwards at the centre of gravity O of this portion making an angle θ with the direction of flow.

It can be resolved in to two componands $mg\cos\theta = dA\cdot dl\,\rho g\cos\theta$ along the direction of flow and $mg\sin\theta = dA\cdot dl\,\rho g\sin\theta$ perpendicular to to the direction of flow.

Let p be the pressure on face A of the tube. Then the pressure on the ~~p~~ face B will be $p + \frac{\partial p}{\partial l}dl$, in the direction shown in fig.

∴ Force action on face A in the forward direction $= p\,dA$

and force acting on face B in the backward direction $= \left(p + \frac{\partial p}{\partial l}\cdot dl\right)dA$

∴ Net force on the mass of the small portion of the fluid is

$$F = p\,dA - \left(p + \frac{\partial p}{\partial l}dl\right)dA + dA\,dl\,\rho g\cos\theta$$

$$\text{or, } F = -\frac{\partial p}{\partial l}dl\,dA + dA\,dl\,\rho g\cos\theta \quad \text{——} (1)$$

Let v be the velocity of the ~~fluid~~ fluid as it enters the tube of flow at A. Then its accelartion is dv/dt

∴ Force acting on the mass $dA\cdot dl\,\rho$ of the fluid in the tube is also given as

$$F = \text{mass} \times \text{accelaration}$$

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$$F = (dA\,dl\,\rho)(dv/dt) \quad \text{—— (2)}$$

Here the velocity is a function of both distance (l) and time t. That is $v = f(l, t)$ therefore

$$dv = \frac{\partial v}{\partial l}\,dl + \frac{\partial v}{\partial t}\cdot dt$$

or $$\frac{dv}{dt} = \frac{\partial v}{\partial l}\cdot\frac{\partial l}{\partial t} + \frac{\partial v}{\partial t}$$

$$\frac{dv}{dt} = v\frac{\partial v}{\partial t} + \frac{\partial v}{\partial t}$$

putting this value in equation (2) we get

$$F = dA\,dl\,\rho\left(v\frac{\partial v}{\partial l} + \frac{\partial v}{\partial t}\right) \quad \text{—— (3)}$$

If h is the vertical height of the tube AB from a chosen plane, then

$$\cos\theta = -\partial h/\partial l$$

From equation (1) and (3)

$$dA\,dl\,\rho\left(v\frac{\partial v}{\partial l} + \frac{\partial v}{\partial t}\right) = -\frac{\partial p}{\partial l}\,dl\,dA - dA\,dl\,\rho g\frac{\partial h}{\partial l}$$

or, $$dA\,dl\,\rho\left(v\frac{\partial v}{\partial l} + \frac{\partial v}{\partial t}\right) = dA\,dl\left(-\frac{\partial p}{\partial l} - \rho g\frac{\partial h}{\partial l}\right)$$

or, $$\boxed{\rho\left(v\frac{\partial v}{\partial l} + \frac{\partial v}{\partial t}\right) = -\frac{\partial p}{\partial l} - \rho g\frac{\partial h}{\partial l}} \quad \text{—— (4)}$$

This is Called Euler's Equation

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